The Analytical Hierarchy Process: A Step-by-Step Approach

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Step 1

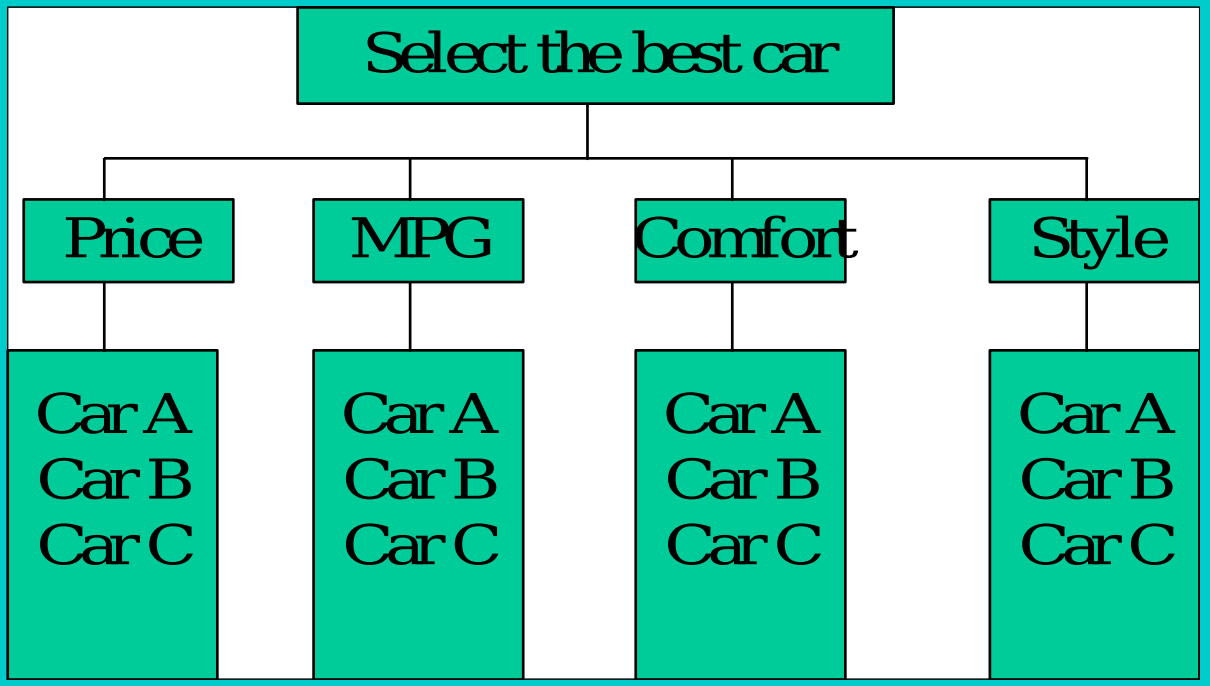
The Analytical Hierarchy Process by Thomas Saaty

A.

	Car A	**Car B**	**Car C**
Price	13100	11200	9500
MPG	18	23	29
Interior	Deluxe	Above Average	Standard
Body	4-Door Mid-size	2-Door Sport	2-door compact
Radio	AM/FM, tape	AM/FM	AM
Engine	6-cylinder	4-cylinder turbo	4-cylinder

Step 2:

B. **Hierarchy of Decisions**



Step 3:

C. Establish Priorities

1. The priorities of the four criteria in terms of the overall goal.
2. The priorities of the three cars in terms of the purchase-price criterion.
3. The priorities of the three cars in terms of the MPG criterion.
4. The priorities of the three cars in terms of the comfort criterion.
5. The priorities of the three cars in terms of the style criterion.

Step 4:

D. Pairwise Comparison Scale

Verbal Judgement of Preference	Numerical Rating
Extremely Preferred	9
Very strong to extremely	8
Very strongly preferred	7
Strongly to very strongly	6
Strongly preferred	5
Moderately to strongly	4
Moderately preferred	3
Equally to moderately	2
Equally preferred	1

Step 5:

E. Pairwise comparison matrix showing preferences for the three cars in terms of comfort

Comfort	Car A	Car B	Car C
Car A	1	2	8
Car B	0.5	1	6
Car C	0.125	0.166666667	1

Step 6:

F. Synthesizing Judgments

Comfort	Car A	Car B	Car C
Car A	1	2	8
Car B	0.5	1	6
Car C	0.125	0.166666667	1
	1.625	3.166666667	15

Sum the columns in the pairwise comparison matrix.

Step 7:

Comfort	Car A	Car B	Car C
Car A	0.615384615	0.631578947	0.533333
Car B	0.307692308	0.315789474	0.4
Car C	0.076923077	0.052631579	0.066667
	1	1	1

Divide elements by the column totals.

Step 9:

Priority Vector for Comfort

Comfort	Car A	Car B	Car C		
Car A	0.615384615	0.631578947	0.533333	0.593432	
Car B	0.307692308	0.315789474	0.4	0.341161	Sum the rows.
Car C	0.076923077	0.052631579	0.066667	0.065407	

Car A	0.593
Car B	0.341
Car C	0.065

The priority vector for the cars with respect to comfort.

Other Pairwise Comparisons

G. Other Pairwise Comparisons

Price	Car A	Car B	Car C	Priority Vecto
Car A	1	0.33333333	0.25	0.123
Car B	3	1	0.5	0.32
Car C	4	2	1	0.557

MPG	Car A	Car B	Car C	
Car A	1	0.25	0.166667	0.087
Car B	4	1	0.333333	0.274
Car C	6	3	1	0.639

Style	Car A	Car B	Car C	
Car A	1	0.33333333	4	0.265
Car B	3	1	7	0.655
Car C	0.25	0.142857143	1	0.08

Step 10:

H. Pairwise comparison matrix for the four criteria in the car selection problem.

Criterion	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG	0.333333333	1	0.25	0.25
Comfort	0.5	4	1	0.5
Style	0.5	4	2	1

I. Priorities for the overall goal.

Price	0.398
MPG	0.085
Comfort	0.218
Style	0.299

Step 11:

J. Developing an overall priority ranking.

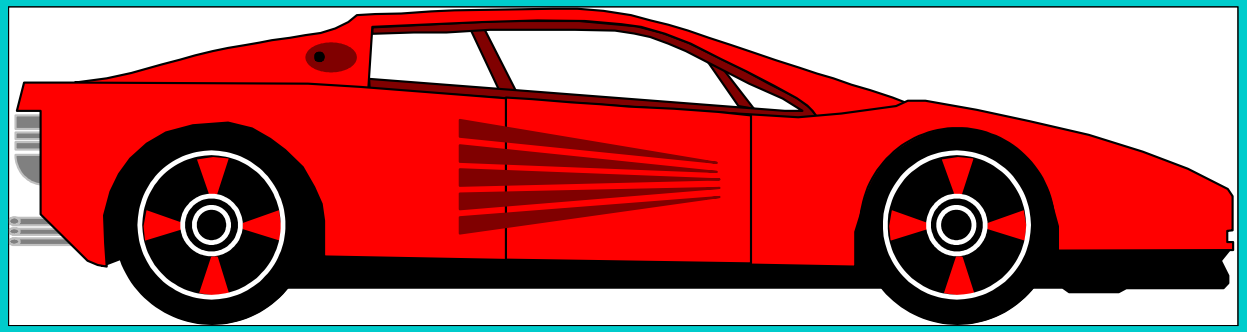
	Criterion	Price	MPG	Comfort	Style
	Car A	0.123	0.087	0.593	0.265
Alternative	**Car B**	0.32	0.274	0.341	0.655
	Car C	0.557	0.639	0.065	0.08

Overall car A priority = .398(0.123)+0.085(0.087)+).218(0.593)+0.299(0.265)=.265
Overall car B priority = .398(0.320)+0.085(0.274)+).218(0.341)+0.299(0.655)=.421
Overall car C priority = .398(0.557)+0.085(0.639)+).218(0.066)+0.299(0.080)=.314

Step 12:

K. Final AHP Ranking of Alternatives

Car B	0.421
Car C	0.314
Car A	0.265
	1



Dr. LaCava's Mid-life Crisis Maserati!